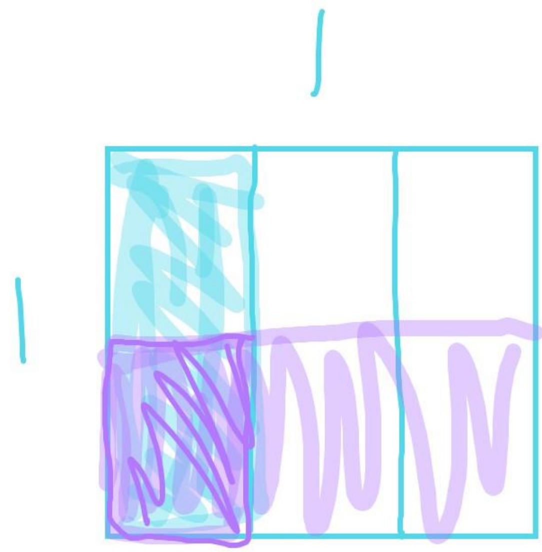
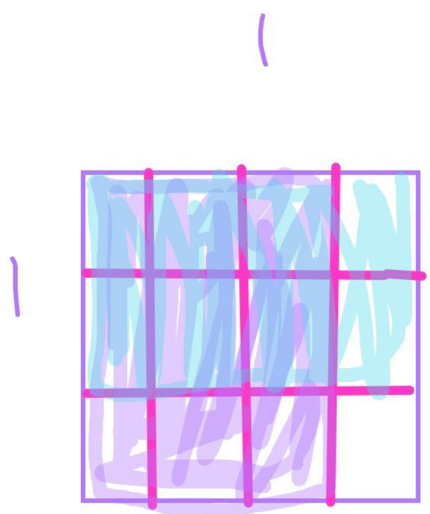
Multiplying Fractions

Here is a lesson I did in December 2023 with a bright 5th grader who was learning how to multiply fractions. The pictures are actual screenshots from our online lesson (used with parents' permission). Most students require several lessons to cover this ground.

Up until this point, multiplication has always led to a bigger answer than the two numbers multiplied. Why does multiplying a fraction often lead to a smaller answer? We started with a visual model to answer this question.



$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

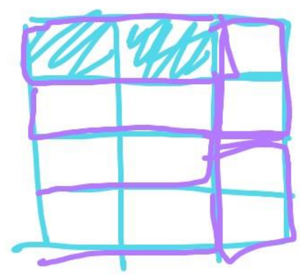


$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$

After the student practiced several problems using the visual model and several problems without, I introduced cross-reducing:

$$\frac{5}{\cancel{6}} \times \frac{\cancel{6}}{11} = \frac{5}{11}$$

$$\frac{30 \div 6}{66 \div 6} = \frac{5}{11}$$



$\frac{1}{4} \times \frac{2}{3}$

$\frac{2}{12} = \frac{1}{6}$

$$\frac{\cancel{3}^{1}}{\cancel{10}_{5}} \times \frac{\cancel{4}^{2}}{\cancel{9}_{3}} = \frac{2}{15}$$

After a bunch of problems just like the above, I upped the difficulty level:

$$\frac{\cancel{3}^{1}}{\cancel{8}_{2}} \times \frac{\cancel{3}^{1}}{\cancel{6}_{2}} \times \frac{\cancel{5}^{1}}{\cancel{5}_{1}} = \frac{1}{4} \times 1 = \frac{1}{4}$$

$$\frac{\cancel{2}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{7}^{1}}{\cancel{105}\,\cancel{5}_{1}} \times \frac{\cancel{20}\,\cancel{4}\,2}{\cancel{3}_{1}} \times \frac{\cancel{5}^{1}}{\cancel{21}\,\cancel{7}_{1}} \times \frac{\cancel{9}\,\cancel{3}^{1}}{\cancel{2}_{1}}$$

$$\frac{1}{1} \times \frac{1}{1} \times \frac{2}{1} \times \frac{1}{1} \times \frac{1}{1} = \frac{2}{1} = 2$$

My student was having fun and loves a challenge, so I threw him this crazy problem:

$$5(1+3+5)$$

$$\frac{\cancel{(31+71+111)}}{(1+3+5)} \times \frac{(5+15+25)}{\cancel{(111+71+31)}}$$

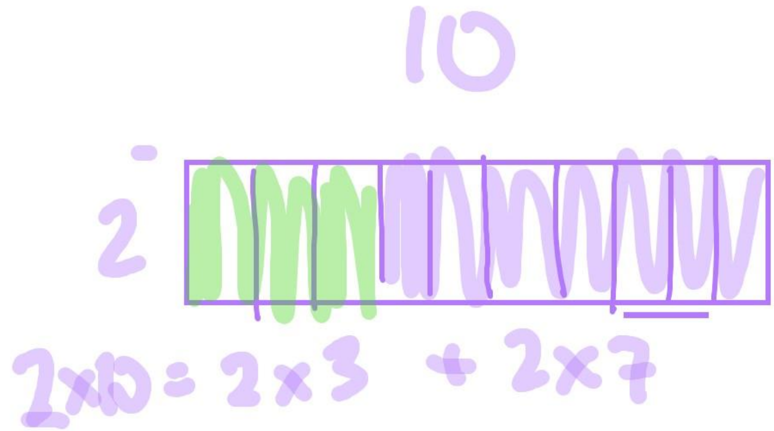
$$\frac{1}{\cancel{9}\,1} \times \frac{\cancel{45}\,5}{1} = \frac{5}{1}$$

The sum on the bottom right is exactly the same as the sum on the top left, just backwards, allowing us to cross both of them out. The sum on the top right is exactly 5 x bigger than the sum on the bottom left, allowing us to pull out a five.

My student needed a refresher on the distributive property at this point, so I went over that:

$$2(3+7)$$

$$2\times3+2\times7$$



It was close to Christmas, so I decided to have a bit of fun. My ulterior motive was to introduce variables (letters) at a young age to smooth the transition to algebra later on.

$$\frac{\cancel{s}a\cancel{n}ta}{\cancel{c}\cancel{la}u\cancel{s}}\times\frac{\cancel{c}aro\cancel{l}}{\cancel{n}ick}=\frac{a^2tro}{uick}$$

“Can we make a Christmas for my brother?” “We sure can!”

$$\frac{\cancel{s}a\cancel{n}\cancel{ta}}{mu\cancel{si}x}\times\frac{hyp\cancel{i}}{t\cancel{an}}=\frac{hapy}{xmus}$$

Still missing an *s* in the denominator!

$$\frac{\text{santa}}{\text{musix}} \times \frac{\text{hypi}}{\text{tans}} = \frac{\text{hapy}}{\text{x-mus}}$$

Got it! A very special Christmas fraction puzzle for my student's brother:

$$\frac{\text{santa}}{\text{musix}} \times \frac{\text{hypi}}{\text{tans}} =$$

At the end, rearrange the letters for a special holiday message :)